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A SIMPLE MODEL FOR STRUCTURAL CONTROL INCLUDING SOIL–STRUCTURE INTERACTION EFFECTS

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SUMMARY

A simple model for the seismic response of a one-storey structure subjected to active control in the presence of soil–structure interaction effects is presented. The approach is based on the successive use of equivalent 1-DOF oscillators which account for the effects of control and soil–structure interaction. Simple expressions for these oscillators based on exact analytical solutions of the control equations and approximate solutions of the interaction equations are presented. The study includes an evaluation of the effects of soil–structure interaction on the seismic response of actively controlled structures in which the control gains have been determined with and without inclusion of soil–structure interaction effects. A simple procedure to include the interaction effects on the control gains is also presented. © 1998 John Wiley & Sons, Ltd.

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KEY WORDS: control; seismic response; soil–structure interaction

INTRODUCTION

In the last few years there has been increased interest in the study of the effects of soil–structure interaction (SSI) on the response of structures subjected to active control.[1–9] These studies follow two lines of inquiry: the first line is concerned with the evaluation of the effects of soil–structure interaction on the response of structures with active control systems designed on the basis of conventional analyses which do not include the interaction effects; the second line considers the incorporation of the soil–structure interaction effects into the design of the control system and algorithms. These two issues were discussed by Wong and Luco[2,3] for a structural model consisting of a shear beam supported on a rigid foundation embedded in a viscoelastic half-space. Control, in this case, was achieved by an absorbing boundary located at the top of the building. Recently, Smith *et al.*[8] and Wu and Smith[9] have considered the effects of SSI on the response of a one-storey actively controlled structure supported on a rigid rectangular foundation resting on an elastic half-space. The analysis was based on the use of external control forces obtained by application of the linear optimal control theory to equations of motion which excluded or included the SSI effects.

Here, we reconsider the problem studied by Smith *et al.*[8] of the effects of SSI on the response of a one-storey structure subjected to active control. The approach differs from that of the previous authors in that elementary methods are used to derive equivalent oscillators which exactly account for the effects of control and approximately account for the effects of soil–structure interaction. By combination of these solutions it is

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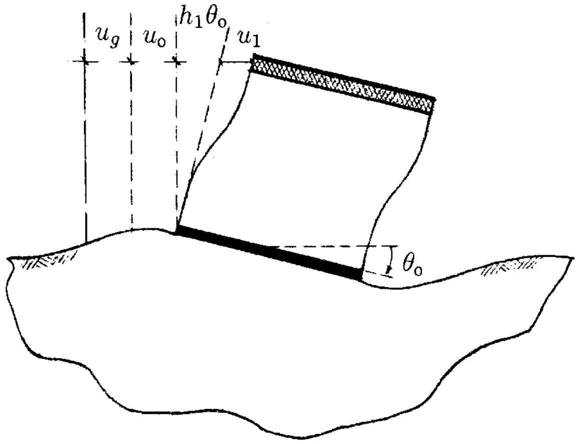

Figure 1. Model of one-storey structure

possible to obtain simple expressions for the seismic response of the one-storey structure including the effects of control and SSI.

We start by summarizing approximate solutions of the interaction equations for a one-storey structure supported on a rigid surface foundation resting on a viscoelastic half-space when subjected to seismic excitation and to external and internal forces. The solution for seismic excitation was initially presented by Bielak[10] and Jennings and Bielak,[11] while that for external forces was presented by Luco *et al.*[12,13]. The well-known result of these approximate solutions is that the deformation of the one-storey structure including SSI effects can be calculated from the response of an equivalent or replacement oscillator supported on a rigid soil and characterized by a modified natural frequency and damping ratio. The equivalent oscillator is subjected to an effective ground acceleration and to an effective force at the top of the structure. The extension of the approximate solution to internal forces (or, more precisely, to internally reacted forces) reveals that the effective force acting on the equivalent oscillator depends on whether the actual forces correspond to internal or external forces. If the forces are internal (such as those on a diagonal tendon system) then they affect the equation controlling the deformation of the superstructure but do not appear in the global equations of motion of the structure–foundation system. On the other hand, external forces appear in the equation reflecting the deformation of the superstructure and in the global equations of motion for the system.

As a second preliminary step, we consider the response of a damped 1-DOF system subjected to active control in the absence of SSI effects. In particular, we consider the optimal control of the free-vibration response of the oscillator for a quadratic performance index. An analytical solution for the control gains is obtained by elementary means which do not require consideration of Riccati's equation. In the particular case of an undamped oscillator, the resulting gains coincide with those obtained by Meirovitch and Öz[14] and Meirovitch and Baruh[15] by analytical solution of Riccati's steady-state equation. Clearly, the response of the oscillator including the effects of control can be assimilated to that of an equivalent (uncontrolled) oscillator with a modified natural frequency and damping. Simple expressions for the frequency and damping ratio of the equivalent oscillator are presented.

In a third stage, we consider the seismic response of a one-storey structure supported on a flexible soil and subjected to active control by internal control forces with gains determined without inclusion of SSI effects. The solution starts by replacing the one-storey superstructure by a modified structure which exactly incorporates the effects of control. The modified one-storey structure is then subjected to the seismic excitation in the presence of soil–structure interaction effects and an approximate solution to the interaction equations is obtained on the basis of an equivalent 1-DOF oscillator which includes both the effects of SSI and control.

Finally, a simple procedure to include the effects of soil–structure interaction on the determination of the control gains is presented. The procedure relies on representing the uncontrolled structure including SSI effects by an equivalent 1-DOF structure on a rigid soil. The optimal control gains for this equivalent structure are then obtained analytically. The original superstructure is then represented by an equivalent one-storey structure which includes the effects of control and the response of this modified structure to seismic excitation in the presence of SSI effects is obtained by the approximate solution summarized in the first part of the study.

EQUIVALENT OSCILLATOR FOR SOIL–STRUCTURE INTERACTION EFFECTS

In here we summarize an approximate solution for the response of a structure subjected to internal and external forces and to seismic excitation when the effects of soil–structure interaction are included in the analysis. The case of seismic excitation was considered by Bielak[10] and Jennings and Bielak,[11] while the case of an external force at the top of the structure was considered by Luco *et al.*[12,13]

We consider the in-plane vibrations of an elastic one-storey structure supported on a flat rigid foundation resting on an elastic half-space. The structure is characterized by its mass m_1, stiffness k_1, damping constant c_1 and height h_1. The foundation is characterized by its mass m_0 and by its equivalent radius a. The underlying elastic half-space is determined by the density ρ, the shear-wave velocity β, the hysteretic damping ratio ξ_s and Poisson's ratio v. The system is subjected to a vertically incident plane SH-wave with total particle motion u_g on the ground surface in absence of the structure, i.e. for free-field conditions. With respect to forces acting on the structure we consider two cases. In the first case, a horizontal external force F_1 is applied at the top of the structure. In a second case, internal horizontal forces F_1 and $F_0 = -F_1$ are applied at the top and base of the structure together with an internal moment $M_0 = -F_1 h_1$ acting on the foundation. This self-balancing set of forces and moments may represent the case of an internal control system such as a tendon system.

The global equations of motion for the structure–foundation system and the equation of motion for the top mass for harmonic excitation with time dependence $e^{i\omega t}$ are

$$m_0 \ddot{u}_0 + m_1(\ddot{u}_0 + h_1 \ddot{\theta}_0 + \ddot{u}_1) + K_H u_0 = F_R - (m_1 + m_0)\ddot{u}_g \tag{1}$$

$$I_0 \ddot{\theta}_0 + h_1 m_1(\ddot{u}_0 + h_1 \ddot{\theta}_0 + \ddot{u}_1) + K_R \theta_0 = M_R - h_1 m_1 \ddot{u}_g \tag{2}$$

$$m_1(\ddot{u}_0 + h_1 \ddot{\theta}_0 + \ddot{u}_1) + c_1 \dot{u}_1 + k_1 u_1 = F_1 - m_1 \ddot{u}_g \tag{3}$$

in which u_0 is the horizontal displacement of the foundation relative to the free-field ground motion, θ_0 is the rocking rotation of the foundation and u_1 is the deformation of the structure. The terms K_H and K_R represent the frequency-dependent horizontal and rocking (complex) impedance fuctions for the foundation and I_0 is the sum of the moments of inertia of the foundation and the top mass with respect to horizontal axes through their centroids. The terms F_R and M_R correspond to the resultant force and the resultant moment (with respect to the base) of all external forces (other than soil reactions) acting on the structure–foundation system. If only an external force F_1 acts at the top of the structure, then $F_R = F_1$ and $M_R = F_1 h_1$. For a system of self-equilibrating internal forces and moments, $F_R = F_1 + F_0 = 0$ and $M_R = F_1 h_1 + M_0 = 0$. In equations (1) and (2) the effects of the coupling impedances $K_{HR} = K_{RH}$ have been neglected.

At this point, we introduce the notation

$$k_1 = \omega_1^2 m_1 \tag{4a}$$

$$c_1 = 2\omega_1 m_1 \xi_1 \tag{4b}$$

$$K_H = m_1 \omega_H^2 \{1 + 2i[\xi_s + (\omega/\omega_H)\xi_H]\} \tag{5a}$$

$$K_R = h_1^2 m_1 \omega_R^2 \{1 + 2i[\xi_s + (\omega/\omega_R)\xi_R]\} \tag{5b}$$

where

$$\omega_1 = \sqrt{k_1/m_1} \tag{6a}$$

$$\xi_1 = \frac{c_1}{2m_1\omega_1} \tag{6b}$$

$$\omega_{\mathrm{H}} = \sqrt{\frac{\mathrm{Re}\,K_{\mathrm{H}}}{m_1}} \tag{7a}$$

$$\xi_{\mathrm{H}} = \frac{1}{2}\left(\frac{\omega_{\mathrm{H}}}{\omega}\right)\left[\left(\frac{\mathrm{Im}\,K_{\mathrm{H}}}{\mathrm{Re}\,K_{\mathrm{H}}}\right) - 2\xi_{\mathrm{s}}\right] \tag{7b}$$

$$\omega_{\mathrm{R}} = \sqrt{\frac{\mathrm{Re}\,K_{\mathrm{R}}}{m_1 h_1^2}} \tag{8a}$$

$$\xi_{\mathrm{R}} = \frac{1}{2}\left(\frac{\omega_{\mathrm{R}}}{\omega}\right)\left[\left(\frac{\mathrm{Im}\,K_{\mathrm{R}}}{\mathrm{Re}\,K_{\mathrm{R}}}\right) - 2\xi_{\mathrm{s}}\right] \tag{8b}$$

The terms ω_1 and ξ_1 correspond to the fixed-base natural frequency and the fixed-base damping ratio for the structure, respectively. The frequencies ω_{H} and ω_{R} correspond to the characteristic frequencies for horizontal and rocking vibrations of a rigid structure on the flexible soil. The damping ratios ξ_{H} and ξ_{R} reflect radiation damping in horizontal and rocking vibrations. The hysteretic material damping in the soil is represented by the terms containing ξ_{s} in equations (5a) and (5b).

By using equation (4) and (5), the equations of motion for the complete system can be rewritten in the symmetric form

$$[D(\omega)]\begin{Bmatrix} (\omega_{\mathrm{H}}/\omega_1)u_0 \\ (\omega_{\mathrm{R}}/\omega_1)h_1\theta_0 \\ u_1 \end{Bmatrix} = \begin{Bmatrix} [\varepsilon F_1/m_1 + \omega^2 u_{\mathrm{g}}]/\omega_1\omega_{\mathrm{H}} \\ [\varepsilon F_1/m_1 + \omega^2 u_{\mathrm{g}}]/\omega_1\omega_{\mathrm{R}} \\ [F_1/m_1 + \omega^2 u_{\mathrm{g}}]/\omega_1^2 \end{Bmatrix} \tag{9}$$

where $\varepsilon = 1$ for the case of an external force F_1 acting at the top of the structure. In the case of a self-equilibrating system of internal forces $\varepsilon = 0$. The elements of the matrix $[D(\omega)]$ are given by

$$D_{11} = \left\{1 - \left(\frac{\omega}{\omega_{\mathrm{H}}}\right)^2 + 2\mathrm{i}\left[\xi_{\mathrm{s}} + \left(\frac{\omega}{\omega_{\mathrm{H}}}\right)\xi_{\mathrm{H}}\right]\right\} \tag{10a}$$

$$D_{22} = \left\{1 - \left(\frac{\omega}{\omega_{\mathrm{R}}}\right)^2 + 2\mathrm{i}\left[\xi_{\mathrm{s}} + \left(\frac{\omega}{\omega_{\mathrm{R}}}\right)\xi_{\mathrm{R}}\right]\right\} \tag{10b}$$

$$D_{33} = \left[1 - \left(\frac{\omega}{\omega_1}\right)^2 + 2\mathrm{i}\left(\frac{\omega}{\omega_1}\right)\xi_1\right] \tag{10c}$$

$$D_{12} = D_{21} = -\left(\frac{\omega}{\omega_{\mathrm{H}}}\right)\left(\frac{\omega}{\omega_{\mathrm{R}}}\right) \tag{10d}$$

$$D_{13} = D_{31} = -\left(\frac{\omega}{\omega_{\mathrm{H}}}\right)\left(\frac{\omega}{\omega_1}\right) \tag{10e}$$

$$D_{23} = D_{32} = -\left(\frac{\omega}{\omega_{\mathrm{R}}}\right)\left(\frac{\omega}{\omega_1}\right) \tag{10f}$$

In writting equation (9), the terms m_0 and I_0 have been ignored when compared with m_1 and $m_1 h_1^2$, respectively.

The solution for the case of a rigid soil (i.e. in the absence of soil–structure interaction) can be obtained by considering the limiting case $\omega_{\mathrm{H}} \to \infty$ and $\omega_{\mathrm{R}} \to \infty$. In this case,

$$u_0 = \theta_0 = 0 \tag{11a}$$

and

$$u_1 = \frac{(\omega/\omega_1)^2 u_{\mathrm{g}} + F_1/(m_1\omega_1^2)}{1-(\omega/\omega_1)^2 + 2\mathrm{i}(\omega/\omega_1)\xi_{\mathrm{s}}} \tag{11b}$$

An approximate solution of equation (9) for the case of an *external* force F_1 acting at the top of the structure ($\varepsilon = 1$) can be obtained by neglecting certain terms involving ξ_1, ξ_{H} and ξ_{R} and assuming that the system frequency $\tilde{\omega}_1$ is close to ω_1, i.e. assuming that the soil–structure interaction effects are small. The resulting solution is given by

$$\left\{\begin{array}{c} u_0 \\ h_1\theta_0 \\ u_1 \end{array}\right\} = \frac{(\frac{\omega}{\tilde{\omega}_1})^2 u_{\mathrm{g}} + (\frac{F_1}{m_1\tilde{\omega}_1^2})}{1-(\frac{\omega}{\tilde{\omega}_1})^2 + 2\mathrm{i}(\frac{\omega}{\tilde{\omega}_1})\tilde{\tilde{\xi}}_1} \left\{\begin{array}{c} (\tilde{\omega}_1/\omega_{\mathrm{H}})^2 \\ (\tilde{\omega}_1/\omega_{\mathrm{R}})^2 \\ (\tilde{\omega}_1/\omega_1)^2 \end{array}\right\} \tag{12}$$

where

$$\frac{1}{\tilde{\omega}_1^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_{\mathrm{H}}^2} + \frac{1}{\omega_{\mathrm{R}}^2} \tag{13}$$

and

$$\tilde{\tilde{\xi}}_1 = \left(\frac{\tilde{\omega}_1}{\omega_1}\right)^3 \xi_1 + \left[1 - \left(\frac{\tilde{\omega}_1}{\omega_1}\right)^2\right] \xi_{\mathrm{s}} + \left(\frac{\tilde{\omega}_1}{\omega_{\mathrm{H}}}\right)^3 \xi_{\mathrm{H}} + \left(\frac{\tilde{\omega}_1}{\omega_{\mathrm{R}}}\right)^3 \xi_{\mathrm{R}} \tag{14}$$

Equations (13) and (14) give approximate expressions for the system frequency $\tilde{\omega}_1$ and for the system damping ratio $\tilde{\tilde{\xi}}_1$.

If we compare the form of the solution for the relative displacement u_1 in equation (12) with the solution for an oscillator on a rigid soil given by equation (11b), we observe that the relative displacement response for an oscillator on a flexible soil can be calculated from the response of an equivalent oscillator on a rigid soil with frequency $\tilde{\omega}_1$ and damping ratio $\tilde{\tilde{\xi}}_1$ subjected to the effective ground motion $\tilde{u}_{\mathrm{g}}$ and to the effective force at the top $\tilde{F}_1$ given, respectively, by

$$\tilde{u}_{\mathrm{g}} = \left(\frac{\tilde{\omega}_1}{\omega_1}\right)^2 u_{\mathrm{g}} \tag{15a}$$

and

$$\tilde{F}_1 = \left(\frac{\tilde{\omega}_1}{\omega_1}\right)^2 F_1 \tag{15b}$$

In the case of a system of *internal* forces (F_1 at the top, $F_0 = -F_1$ at the base, and $M_0 = -F_1 h_1$) the parameter $\varepsilon = 0$ and the approximate solution is given by

$$\left\{\begin{array}{c} u_0 \\ h_1\theta_0 \\ u_1 \end{array}\right\} = \frac{(\frac{\omega}{\tilde{\omega}_1})^2 u_{\mathrm{g}} + (\frac{\tilde{\omega}_1}{\omega_1})^2(\frac{F_1}{m_1\tilde{\omega}_1^2})}{1-(\frac{\omega}{\tilde{\omega}_1})^2 + 2\mathrm{i}(\frac{\omega}{\tilde{\omega}_1})\tilde{\tilde{\xi}}_1} \left\{\begin{array}{c} (\tilde{\omega}_1/\omega_{\mathrm{H}})^2 \\ (\tilde{\omega}_1/\omega_{\mathrm{R}})^2 \\ (\tilde{\omega}_1/\omega_1)^2 \end{array}\right\} \tag{16}$$

in which $\tilde{\omega}_1$ and $\tilde{\xi}_1$ are also given by equations (13) and (14). In this case of internal forces ($\varepsilon = 0$), the equivalent oscillator on a rigid soil is subjected to the effective ground motion $\tilde{u}_g$ and to the effective force at the top $\tilde{F}_1$ given, respectively, by

$$\tilde{u}_g = \left(\frac{\tilde{\omega}_1}{\omega_1}\right)^2 u_g \tag{17a}$$

and

$$\tilde{F}_1 = \left(\frac{\tilde{\omega}_1}{\omega_1}\right)^4 F_1 \tag{17b}$$

It must be noted that the equivalent force at the top $\tilde{F}_1$ given by equation (17b) differs from that given by equation (15b).

For later use, we note that the impedance functions K_H and K_R are usually normalized in the form

$$K_H = \rho\beta^2 a(k_H + \mathrm{i}a_0 c_H) \tag{18a}$$

$$K_R = \rho\beta^2 a^3(k_R + \mathrm{i}a_0 c_R) \tag{18b}$$

where a is the equivalent radius of the foundation, $a_0 = \omega a/\beta$ is a dimensionless frequency, k_H and k_R are the normalized horizontal and rocking stiffness coefficients, and c_H and c_R are the normalized horizontal and rocking damping coefficients. In general, the coefficients k_H, c_H, k_R and c_R depend on a_0, the soil damping ratio ξ_s and Poisson's ratio v. To determine $\tilde{\omega}_1$ and $\tilde{\xi}_1$ by using equations (13) and (14) these coefficients should be calculated at $\tilde{a}_0 = \tilde{\omega}_1 a/\beta$ and, consequently, an iterative process is required to determine $\tilde{\omega}_1$.

Substitution from equations (16) into equations (7) and (8) leads to

$$\frac{\omega_1}{\omega_H} = \left(\frac{\omega_1 a}{\beta}\right)\sqrt{\frac{(m_1/\rho a^3)}{k_H}} \tag{19a}$$

$$\xi_H = \frac{c_H - 2\xi_s k_H/a_0}{2\sqrt{(m_1/\rho a^3)k_H}} \tag{19b}$$

$$\frac{\omega_1}{\omega_R} = \left(\frac{\omega_1 a}{\beta}\right)\left(\frac{h_1}{a}\right)\sqrt{\frac{(m_1/\rho a^3)}{k_R}} \tag{20a}$$

$$\xi_H = \frac{c_R - 2\xi_s k_R/a_0}{2(h_1/a)\sqrt{(m_1/\rho a^3)k_R}} \tag{20b}$$

where $(m_1/\rho a^3)$ is the mass ratio and h_1/a is the slenderness ratio.

The variations of the system frequency $\tilde{\omega}_1$ normalized by the fixed-base frequency ω_1 and of the system damping ratio $\tilde{\xi}_1$ as a function of the relative stiffness parameter $a_1 = \omega_1 a/\beta$ are shown in Figure 2 for two values of the slenderness parameter h_1/a. The results are based on a structure with a mass ratio $m_1/\rho a^3 = 1{\cdot}0$ and with a fixed-base damping ratio $\xi_1 = 0{\cdot}02$. The circular foundation of radius a rests on a viscoelastic half-space characterized by the shear-wave velocity β, Poisson's ratio $v = 1/3$ and material damping ratio $\xi_s = 0{\cdot}02$ (it was assumed that the damping ratios for P- and S-waves were equal to ξ_s). The impedance functions for a circular foundation resting on an elastic half-space presented by Luco and Mita[16] were used in the analysis after modification to account for the effects of material damping.

The information in Figure 2 reflects the well-known results that the system frequency is reduced and the system damping is generally increased as the soil becomes softer, i.e. as a_1 increases. The reduction of the system frequency is larger for the more slender structure while the increase in the system damping is smaller. (Indeed, if $\xi_s = 0$, the system damping ratio $\tilde{\xi}_1$ may be smaller than ξ_1 if the structure is very slender.)

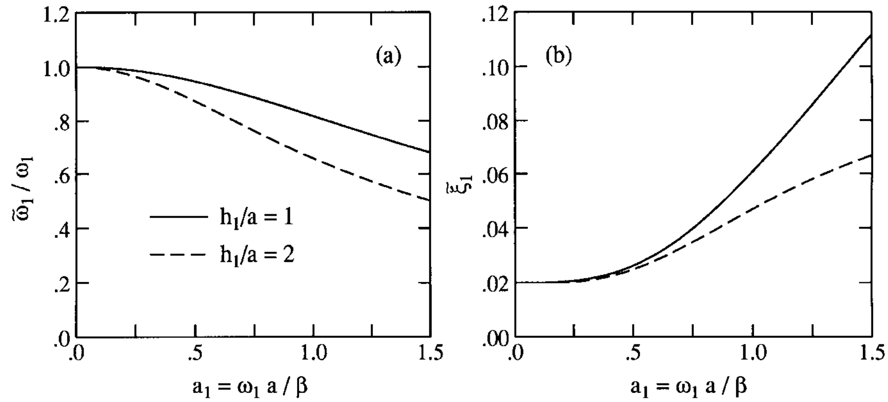

Figure 2. Normalized system frequency $\tilde{\omega}_1/\omega_1$ and system damping ratio $\tilde{\xi}_1$ as a function of the relative stiffness ratio $a_1 = \omega_1 a/\beta$, ($m_1/\rho a^3 = 0{\cdot}5$, $h_1/a = 1$ and 2, $\xi_1 = 0{\cdot}02$, $\xi_s = 0{\cdot}02$, $\nu = 1/3$)

OPTIMAL CONTROL OF A SINGLE-DEGREE-OF-FREEDOM OSCILLATOR

We consider the optimal control of the free vibrations of a single-degree-of-freedom oscillator characterized by the mass m_1, the frequency ω_1 and by the damping ratio ξ_1. The equation of motion of the oscillator subjected to the control force $F_1(t)$ is given by

$$\ddot{u}_1 + 2\xi_1\omega_1\dot{u}_1 + \omega_1^2 u_1 = F_1(t)/m_1 \tag{21}$$

with appropriate initial conditions on $u_1(0)$ and $\dot{u}_1(0)$. The control force is selected in such a way that the quadratic performance index J, defined as

$$J = \int_0^\infty \{\gamma_1\dot{u}_1^2 + \gamma_2\omega_1^2 u_1^2 + r[F_1(t)/m_1]^2\}\,\mathrm{d}t \tag{22}$$

is minimized subject to the constraining equation (21). In equation (22), γ_1, γ_2 and r are weighting factors. In particular, if $\gamma_1 = \gamma_2$ then the first two terms in the integrand are proportional to the sum of the kinetic and strain energy of the oscillator. The factor r is a measure of the cost of the control force.

Next, we write the control force $F_1(t)$ in the form

$$F_1(t) = -m_1[\omega_1^2 g u_1 + \omega_1 h\dot{u}_1] \tag{23}$$

where g and h are the normalized gains. Substitution from equation (23) into the performance index leads to

$$J = (\gamma_1 + \beta h^2)\int_0^\infty \dot{u}_1^2\,\mathrm{d}t + \omega_1^2(\gamma_2 + \beta g^2)\int_0^\infty u_1^2\,\mathrm{d}t - \omega_1\beta g h u_1^2(0) \tag{24}$$

in which $\beta = r\omega_1^2$.

Also, substitution from equation (23) into equation (21) allows us to write the equation of motion for the controlled oscillator in the form

$$\ddot{u}_1 + 2\xi_{1c}\omega_{1c}\dot{u}_1 + \omega_{1c}^2 u_1 = 0 \tag{25}$$

which corresponds to the equation of motion for free vibrations of an equivalent oscillator of mass $m_{1c} = m_1$, frequency

$$\omega_{1c} = \omega_1\sqrt{1+g} \tag{26}$$

and damping ratio

$$\xi_{1c} = (2\xi_1 + h)/2\sqrt{1+g} \tag{27}$$

Multiplying equation (25) by $\dot{u}_1(t)$ and $u_1(t)$ and integrating leads to

$$\int_0^\infty \dot{u}_1^2\,\mathrm{d}t = [\dot{u}_1^2(0) + \omega_{1c}^2 u_1^2(0)]/(4\xi_{1c}\omega_{1c}) \tag{28}$$

and

$$\int_0^\infty u_1^2\,\mathrm{d}t = \left[\int_0^\infty \dot{u}_1^2\,\mathrm{d}t + u_1(0)\dot{u}_1(0) + \xi_{1c}\omega_{1c}u_1^2(0)\right] \Big/ \omega_{1c}^2 \tag{29}$$

which upon substitution into equation (24) results in the following expression for the performance index

$$\begin{aligned}\omega_1 J(g,h) = &\left[\frac{\gamma_1 + \beta h^2 + G(g)}{2(2\xi_1 + h)}\right][\dot{u}_1^2(0) + \omega_1^2 u_1^2(0)] \\ &+ G(g)\omega_1 u_1(0)\dot{u}_1(0) + \frac{1}{2}G(g)(2\xi_1 + h)\omega_1^2 u_1^2(0) \\ &+ g\left[\frac{\gamma_1 + \beta h^2 + G(g)}{2(2\xi_1 + h)} - \beta h\right]\omega_1^2 u_1^2(0)\end{aligned} \tag{30}$$

where

$$G(g) = \frac{\gamma_2 + \beta g^2}{1+g} \tag{31}$$

The optimal values for the gains g and h which minimize $J(g,h)$ are then obtained by setting the partial derivatives $\partial J/\partial g$ and $\partial J/\partial h$ equal to zero. It is found that the optimal gains satisfy the conditions

$$\frac{\mathrm{d}G}{\mathrm{d}g} = \frac{2\beta g}{1+g} - \frac{\gamma_2 + \beta g^2}{(1+g)^2} = 0 \tag{32}$$

and

$$\frac{\gamma_1 + \beta h^2 + G(g)}{2(2\xi_1 + h)} - \beta h = 0 \tag{33}$$

The resulting optimal gains are given by

$$g = \sqrt{1 + \alpha\gamma_2} - 1 \tag{34}$$

$$h = \sqrt{4\xi_1^2 + \alpha\gamma_1 + 2\sqrt{1+\alpha\gamma_2} - 2} - 2\xi_1 \tag{35}$$

where $\alpha = 1/\beta = 1/(\omega_1^2 r)$ is the control parameter (the uncontrolled case corresponds to $\alpha = 0$). For the special case of $\gamma_1 = \gamma_2 = 1$ and $\xi_1 = 0$ the present results coincide with those obtained by Meirovitch and Öz[14] and Meirovitch and Baruh[15] by a procedure involving the solution of a Riccati equation.

The value of the performance index for the optimal gains is given by

$$J = \{h\dot{u}_1^2(0) + 2g\omega_1 u_1(0)\dot{u}_1(0) + [h + g(2\xi_1 + h)]\,\omega_1^2 u_1^2(0)\}/\alpha\omega_1 \tag{36}$$

which can be confirmed to be a positive-definite form of $u_1(0)$ and $\dot{u}_1(0)$.

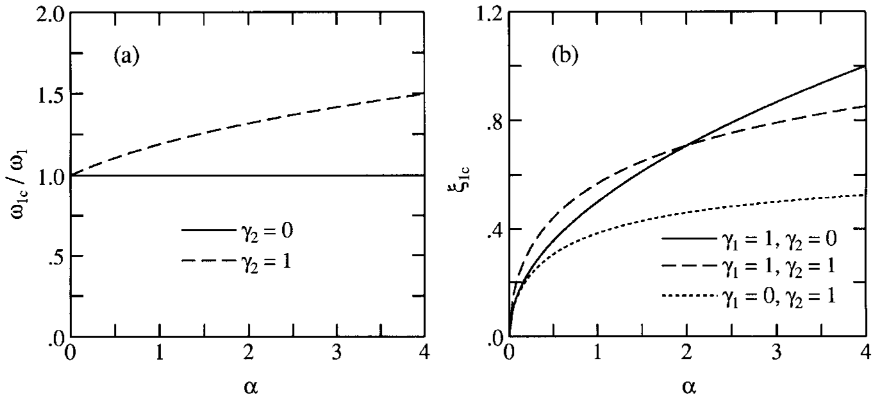

Figure 3. Normalized frequency ω_{1c}/ω_1 and damping ratio ξ_{1c} of the equivalent oscillator as a function of $\alpha = (\omega_1^2 r)^{-1}$ for different values of the weights γ_1 and γ_2 $(\xi_1 = 0)$

Finally, substitution from equations (34) and (35) into equations (26) and (27) leads to the following expressions for the frequency ω_{1c} and damping ratio ξ_{1c} of the equivalent oscillator:

$$\omega_{1c} = \omega_1(1 + \alpha\gamma_2)^{1/4} \tag{37}$$

$$\xi_{1c} = \left\{ \frac{\xi_1^2}{\sqrt{1 + \alpha\gamma_2}} + \frac{1}{2}\left[1 - \frac{(1 - \alpha\gamma_1/2)}{\sqrt{1 + \alpha\gamma_2}}\right] \right\}^{1/2} \tag{38}$$

Some of the characteristics of the equivalent oscillator defined by equations (37) and (38) need to be stated here:

(i) in the uncontrolled case $(\alpha = 0)$ $\omega_{1c} = \omega_1$ and $\xi_{1c} = \xi_1$,
(ii) the equivalent frequency ω_{1c} is independent of γ_1 — the weighting factor for the velocity $\dot{u}_1$,
(iii) if $\gamma_2 = 0$, i.e. if the displacement u_1 is not included in the performance index, then $\omega_{1c} = \omega_1$ and only the effective damping ratio changes to $\xi_{1c} = [\xi_1^2 + (\alpha\gamma_1/4)]^{1/2}$, and
(iv) if $\gamma_1 = 0$, i.e. if the velocity $\dot{u}_1$ does not appear in the performance index, then $\xi_{1c} \to 1/\sqrt{2}$ as $\alpha \to \infty$ (i.e. as the cost r of control tends to zero).

The variations of ω_{1c}/ω_1 and ξ_{1c} against the control parameter $\alpha = 1/(\omega_1^2 r)$ are shown in Figure 3 for different values of the weighting factors γ_1 and γ_2. The results in Figure 3(b) correspond to the case $\xi_1 = 0$. The controlled frequency ω_{1c} remains unchanged (for $\gamma_2 = 0$) or increases slightly with α while the equivalent damping ratio ξ_{1c} increases strongly with α. For example, for $\gamma_1 = \gamma_2 = 1{\cdot}0$, $\xi_1 = 0$ and $\alpha = 2$, the frequency ω_{1c} increases to $1{\cdot}32\omega_1$ while the damping ratio ξ_{1c} becomes equal to $0{\cdot}707$.

RESPONSE OF A CONTROLLED 1-DOF OSCILLATOR WITH GAINS DETERMINED WITHOUT SSI EFFECTS

In this section we consider the effects of soil–structure interaction on the seismic response of an actively controlled structure in which the gains have been determined without consideration of the interaction effects. For this purpose we consider a one-storey structure characterized by the parameters m_1, ω_1, ξ_1 and h_1 and subjected to an *internal* control force $F_1(t)$ acting on the top mass. It is assumed that the control force is given by equation (23) with the control gains g and h determined by minimizing the performance index given by equation (22) for free vibrations of the structure with the foundation kept fixed. The resulting gains are given by equations (34) and (35).

When the effects of soil–structure interaction are included, the equation of motion for the top mass is given by equation (3) in which the control force F_1 is given by equation (23). The resulting equation of motion for harmonic vibrations can be written in the form

$$m_1(\ddot{u}_0 + h_1\ddot{\theta}_0 + \ddot{u}_1) + 2m_1\omega_{1c}\xi_{1c}\dot{u}_1 + m_1\omega_{1c}^2 u_1 = -m_1\ddot{u}_g \tag{39}$$

in which ω_{1c} and ξ_{1c} are the effective controlled frequency and damping ratio given by equations (37) and (38), respectively.

The global equations of motion for the structure–foundation system are given by equations (1) and (2) with $F_R = M_R = 0$ ($\varepsilon = 0$) since the control forces in this case are internal forces. The system of interaction equations for u_0, θ_0 and u_1 corresponds to

$$[D_c(\omega)]\left\{\begin{matrix}(\omega_H/\omega_{1c})u_0\\(\omega_R/\omega_{1c})h_1\theta_0\\u_1\end{matrix}\right\} = \left(\frac{\omega}{\omega_{1c}}\right)^2 u_g\left\{\begin{matrix}(\omega_{1c}/\omega_H)\\(\omega_{1c}/\omega_R)\\1\end{matrix}\right\} \tag{40}$$

in which the matrix $[D_c(\omega)]$ is given by equation (10) after the substitutions $\omega_1 \to \omega_{1c}$ and $\xi_1 \to \xi_{1c}$.

The approximate solution to equation (40) is given by

$$\left\{\begin{matrix}u_0\\h_1\theta_0\\u_1\end{matrix}\right\} = \frac{\left(\dfrac{\omega}{\tilde{\omega}_{1c}}\right)^2 u_g}{1 - \left(\dfrac{\omega}{\tilde{\omega}_{1c}}\right)^2 + 2\mathrm{i}\left(\dfrac{\omega}{\tilde{\omega}_{1c}}\right)\tilde{\xi}_{1c}}\left\{\begin{matrix}(\tilde{\omega}_{1c}/\omega_H)^2\\(\tilde{\omega}_{1c}/\omega_R)^2\\(\tilde{\omega}_{1c}/\omega_{1c})^2\end{matrix}\right\} \tag{41}$$

where

$$\frac{1}{\tilde{\omega}_{1c}^2} = \frac{1}{\omega_{1c}^2} + \frac{1}{\omega_H^2} + \frac{1}{\omega_R^2} \tag{42}$$

and

$$\tilde{\xi}_{1c} = \left(\frac{\tilde{\omega}_{1c}}{\omega_{1c}}\right)^3 \xi_{1c} + \left[1 - \left(\frac{\tilde{\omega}_{1c}}{\omega_{1c}}\right)^2\right]\xi_s + \left(\frac{\tilde{\omega}_{1c}}{\omega_H}\right)^3 \xi_H + \left(\frac{\tilde{\omega}_{1c}}{\omega_R}\right)^3 \xi_R \tag{43}$$

The peak amplitudes of the transfer functions u_0/u_g, $h_1\theta_0/u_g$ and u_1/u_g at $\omega = \tilde{\omega}_{1c}$ are given by

$$|u_0/u_g| = \frac{(\tilde{\omega}_{1c}/\omega_H)^2}{2\tilde{\xi}_{1c}} \tag{44a}$$

$$|h_1\theta_0/u_g| = \frac{(\tilde{\omega}_{1c}/\omega_R)^2}{2\tilde{\xi}_{1c}} \tag{44b}$$

$$|u_1/u_g| = \frac{(\tilde{\omega}_{1c}/\omega_{1c})^2}{2\tilde{\xi}_{1c}} \tag{44c}$$

The corresponding amplitude of the control force at $\omega = \tilde{\omega}_{1c}$ is given by

$$|F_1/\omega_1^2 m_1 u_g| = \sqrt{g^2 + \left(\frac{\tilde{\omega}_{1c}}{\omega_1}\right)^2 h^2}\left[\frac{(\tilde{\omega}_{1c}/\omega_{1c})^2}{2\tilde{\xi}_{1c}}\right] \tag{45}$$

To illustrate the effects of soil–structure interaction and control on the seismic response of a one-storey structure, we consider a structure characterized by $m_1/\rho a^3 = 1{\cdot}0$, $h_1/a = 1$ or $h_1/a = 2$ and $\xi_1 = 0{\cdot}02$. The foundation is modelled as a flat rigid disk foundation of radius a placed on a uniform half-space characterized

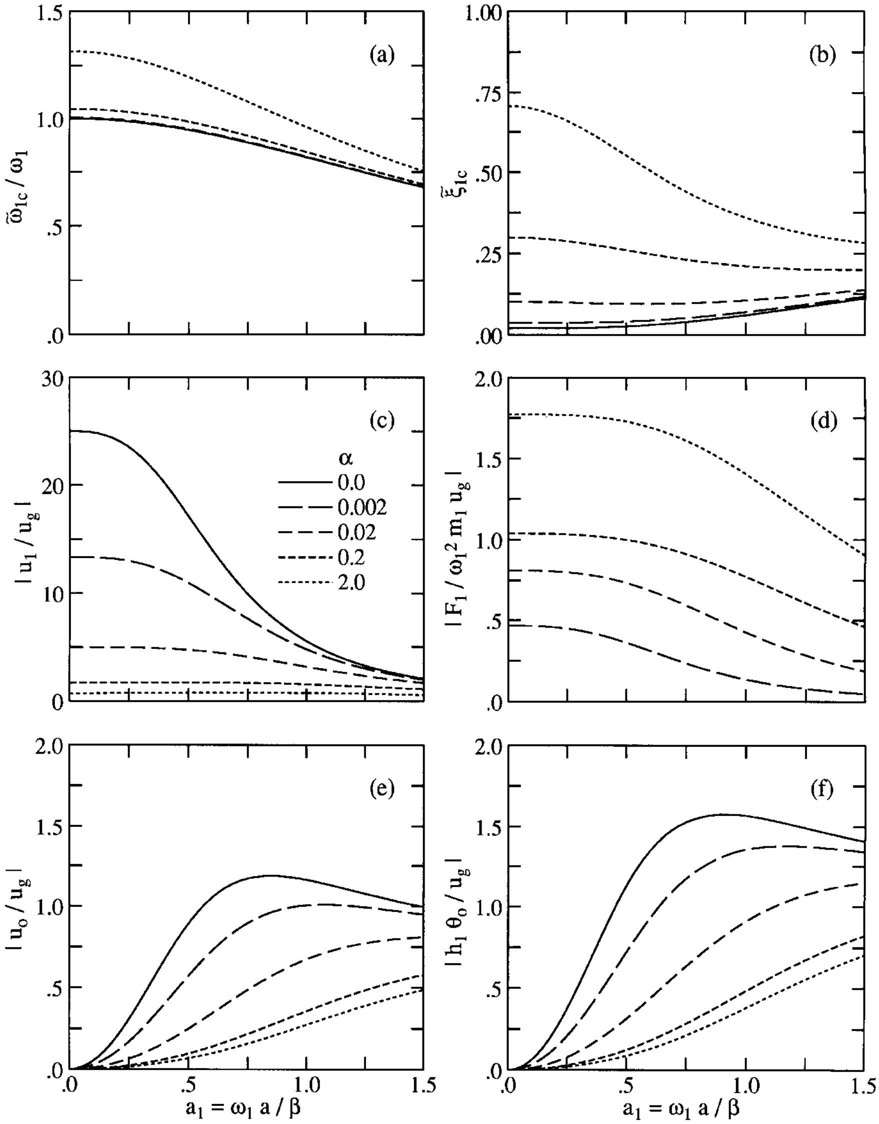

Figure 4. Effects of SSI on the response of an actively controlled one-storey structure for the case of gains determined without SSI. Results include: (a) $\tilde{\omega}_{1c}/\omega_1$, (b) $\tilde{\xi}_{1c}$ and (c) $|u_1/u_g|$, (d) $|F_1/\omega_1^2 m_1 u_g|$, (e) $|u_0/u_g|$ and (f) $|h_1\theta_0/u_g|$ at $\omega = \tilde{\omega}_{1c}$. Slenderness ratio $h_1/a = 1{\cdot}0$

by the shear-wave velocity β, density ρ, Poisson's ratio $v = 1/3$ and hysteretic material damping ratio $\xi_s = 0{\cdot}02$. The control gains are determined for weighting factors of $\gamma_1 = \gamma_2 = 1{\cdot}0$ and do not include SSI effects. The results obtained are shown in Figures 4 and 5. In particular, each figure shows the apparent system frequency $\tilde{\omega}_{1c}$ including the effects of SSI and control and the corresponding system damping ratio $\tilde{\xi}_{1c}$. Also shown are the amplitudes of the transfer functions u_1/u_g, $F_1/\omega_1^2 m_1 u_g$, u_0/u_g and $h_1\theta_0/u_g$ at the frequency $\tilde{\omega}_{1c}$. The results in Figures 4 and 5 are presented versus the relative stiffness parameter $a_1 = \omega_1 a/\beta$ where ω_1 is the fixed-base natural frequency of the uncontrolled superstructure, a is the radius of the foundation and β is the shear-wave velocity of the soil. The results are shown for several values of the control parameter $\alpha = (\omega_1^2 r)^{-1}$ ranging from $\alpha = 0$ for the uncontrolled case to $\alpha = 2$ in which there is a large control force.

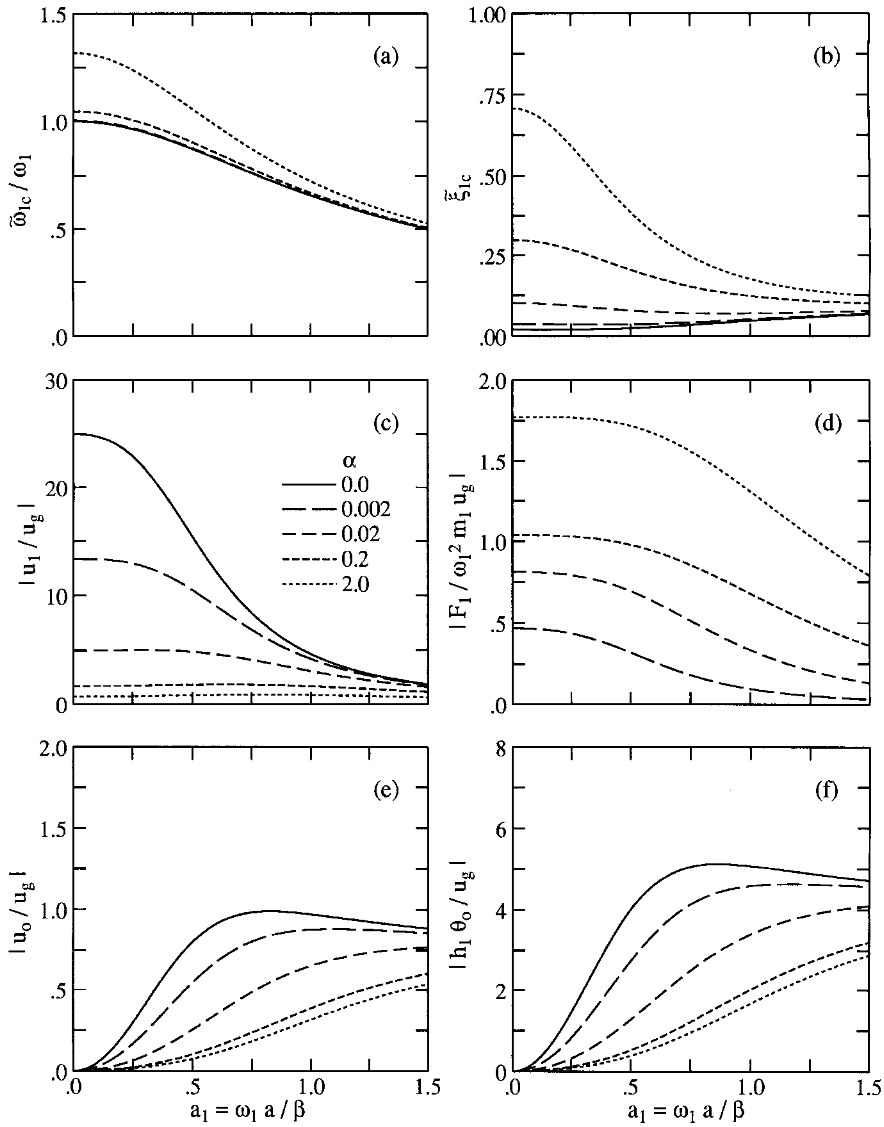

Figure 5. Effects of SSI on the response of an actively controlled one-storey structure for the case of gains determined without SSI. Results include: (a) $\tilde{\omega}_{1c}/\omega_1$, (b) $\tilde{\xi}_{1c}$ and (c) $|u_1/u_g|$, (d) $|F_1/\omega_1^2 m_1 u_g|$, (e) $|u_0/u_g|$ and (f) $|h_1\theta_0/u_g|$ at $\omega = \tilde{\omega}_{1c}$. Slenderness ratio $h_1/a = 2{\cdot}0$

The results in Figures 4 and 5 indicate that as control increases (i.e. as α increases) the system frequency $\tilde{\omega}_{1c}$, the system damping ratio $\tilde{\xi}_{1c}$ and the control force F_1 also increase while the deformation of the structure, the relative displacement of the base and the rocking of the base decrease significantly.

The effects of soil–structure interaction tend to reduce the system frequency $\tilde{\omega}_{1c}$ and the control force F_1 but the reductions are significant only for values of $a_1 > 0{\cdot}25$. The interaction effects clearly increase the relative displacement and rocking motion of the base. The effects of SSI on the deformation of the structure depend on the amount of control acting on the structure. For a small amount of control ($\alpha < 0{\cdot}02$), the system damping ratio $\tilde{\xi}_{1c}$ increases and the deformation of the structure u_1 decreases significantly as the soil becomes softer. On the other hand, if the amount of control is large ($\alpha > 0{\cdot}02$), the system damping ratio $\tilde{\xi}_{1c}$ decreases with a_1 and a slight increase in the deformation u_1 of the structure can be obtained as the soil becomes softer. For

fixed values of $a_1 = \omega_1 a/\beta$, $m_1/\rho a^3$ and α, the effects of interaction appear to be stronger as the slenderness ratio h_1/a increases.

RESPONSE OF A CONTROLLED 1-DOF OSCILLATOR WITH GAINS DETERMINED INCLUDING SSI EFFECTS

We consider now the seismic response of a one-storey structure subjected to active control in which the gains have been determined by approximately including the effects of soil–structure interation. To start, we recall from equation (16) that the relative displacement u_1 for a one-storey structure supported on a flexible soil and subjected to an *internal* force F_1 and to the seismic excitation u_g can be approximately obtained as the response of an equivalent 1-DOF structure on a rigid soil satisfying the equation of motion

$$\ddot{u}_1 + 2\tilde{\xi}_1\tilde{\omega}_1\dot{u}_1 + \tilde{\omega}_1^2 u_1 = \tilde{F}_1/m_1 - \ddot{\tilde{u}}_g \tag{46}$$

in which the equivalent frequency $\tilde{\omega}_1$ and damping ratio $\tilde{\xi}_1$ are given by equations (13) and (14), respectively. The corresponding effective input ground motion $\tilde{u}_g$ and the effective force $\tilde{F}_1$ are given by equations (17a) and (17b), respectively.

To derive the optimal control parameters for the equivalent oscillator we use again, for the purpose of comparison, the same performance index J,

$$J = \int_0^\infty \left\{\gamma_1\dot{u}_1^2 + \gamma_2\omega_1^2 u_1^2 + r[F_1/m_1]^2\right\} \mathrm{d}t \tag{47}$$

which we rewrite in the form

$$J = \int_0^\infty \left\{\gamma_1\dot{u}_1^2 + \tilde{\gamma}_2\tilde{\omega}_1^2 u_1^2 + \tilde{r}[\tilde{F}_1/m_1]^2\right\}^2 \mathrm{d}t \tag{48}$$

in which

$$\tilde{\gamma}_2 = (\omega_1/\tilde{\omega}_1)^2\gamma_2 \tag{49}$$

and

$$\tilde{r} = (\omega_1/\tilde{\omega}_1)^8 r \tag{50}$$

To simplify the process of obtaining the optimal gains, the displacement u_1 is constrained to satisfy the equation of motion for the equivalent oscillator in the absence of seismic excitation.

Following equations (23), (34) and (35), the optimal effective control force is given by

$$\tilde{F}_1 = -m_1[\tilde{\omega}_1^2\bar{g}u_1 + \tilde{\omega}_1\bar{h}\dot{u}_1] \tag{51}$$

in which $\bar{g}$ and $\bar{h}$ are the effective gains

$$\bar{g} = \sqrt{1+\tilde{\alpha}\tilde{\gamma}_2} - 1 \tag{52}$$

$$\bar{h} = \sqrt{4\tilde{\xi}_1^2 + \tilde{\alpha}\gamma_1 + 2\sqrt{1+\tilde{\alpha}\tilde{\gamma}_2} - 2} - 2\tilde{\xi}_1 \tag{53}$$

where $\tilde{\alpha} = 1/(\tilde{\omega}_1^2\tilde{r}) = (\tilde{\omega}_1/\omega_1)^6\alpha$. The actual control force $F_1 = (\omega_1/\tilde{\omega}_1)^4\tilde{F}_1$ is given by

$$F_1(t) = -m_1[\omega_1^2 g u_1 + \omega_1 h\dot{u}_1] \tag{54}$$

where the actual gains, including SSI, are

$$g = (\omega_1/\tilde{\omega}_1)^2\bar{g} \tag{55}$$

$$h = (\omega_1/\tilde{\omega}_1)^3\bar{h} \tag{56}$$

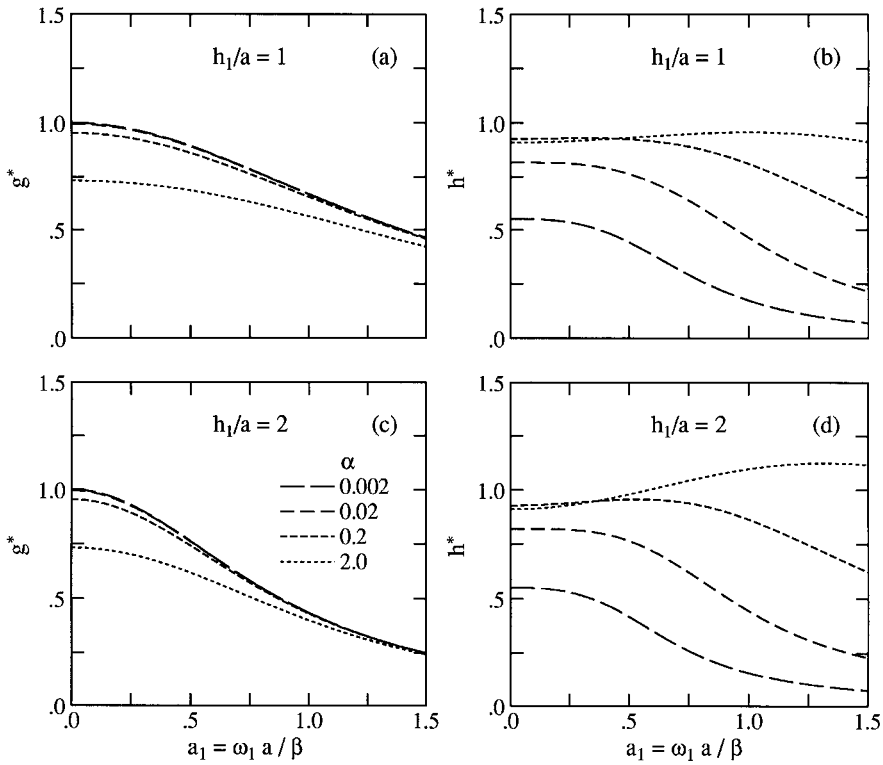

Figure 6. Effects of SSI on the normalized control gains $g^* = (2g/\alpha)$ and $h^* = (h/\sqrt{2\alpha})$ for $h_1/a = 1$ (a, b) and $h_1/a = 2$ (c, d)

The effects of SSI on the gains g and h are illustrated in Figure 6 for the case $m_1/\rho a^3 = 1{\cdot}0$, $h_1/a = 1$ (a, b) and $h_1/a = 2$ (c, d), $\xi_1 = 0{\cdot}02$, $\xi_s = 0{\cdot}02$, $v = 1/3$, $\gamma_1 = 1$ and $\gamma_2 = 1$. In this figure, the values of $g^* = (2g/\alpha)$ and $h^* = (h/\sqrt{2\alpha})$ are shown versus the relative stiffness parameter $a_1 = \omega_1 a/\beta$ for values of the control parameter $\alpha = (\omega_1^2 r)^{-1}$ of 0·002, 0·02, 0·2 and 2·0. It is apparent that the effects of SSI on g and h are significant only if $a_1 > 0{\cdot}5$. The effects appear to be stronger for the more slender structure and when the control force is small.

To calculate the relative response of the controlled structure we have several possibilities. One approximation is to substitute from equation (51) into equation (46) leading to

$$\ddot{u}_1 + 2\tilde{\tilde{\xi}}_{1c}\tilde{\omega}_{1c}\dot{u}_1 + \tilde{\omega}_{1c}^2 u_1 = -\ddot{\tilde{u}}_g \tag{57}$$

in which now

$$\tilde{\omega}_{1c} = \tilde{\omega}_1(1 + \tilde{\alpha}\tilde{\gamma}_2)^{1/4} \tag{58}$$

and

$$\tilde{\tilde{\xi}}_{1c} = \left\{ \frac{\tilde{\tilde{\xi}}_1^2}{\sqrt{1 + \tilde{\alpha}\tilde{\gamma}_2}} + \frac{1}{2}\left[1 - \frac{(1 - \tilde{\alpha}\gamma_1/2)}{\sqrt{1 + \tilde{\alpha}\tilde{\gamma}_2}}\right] \right\}^{1/2} \tag{59}$$

In the frequency domain, the peak amplitude of the transfer function u_1/u_g for the oscillator defined by equations (58) and (59) would occur at $\omega = \tilde{\omega}_{1c}$ and would be given by

$$\left|\frac{u_1}{u_g}\right| = \frac{(\tilde{\omega}_1/\omega_1)^2}{2\tilde{\tilde{\xi}}_{1c}} \tag{60}$$

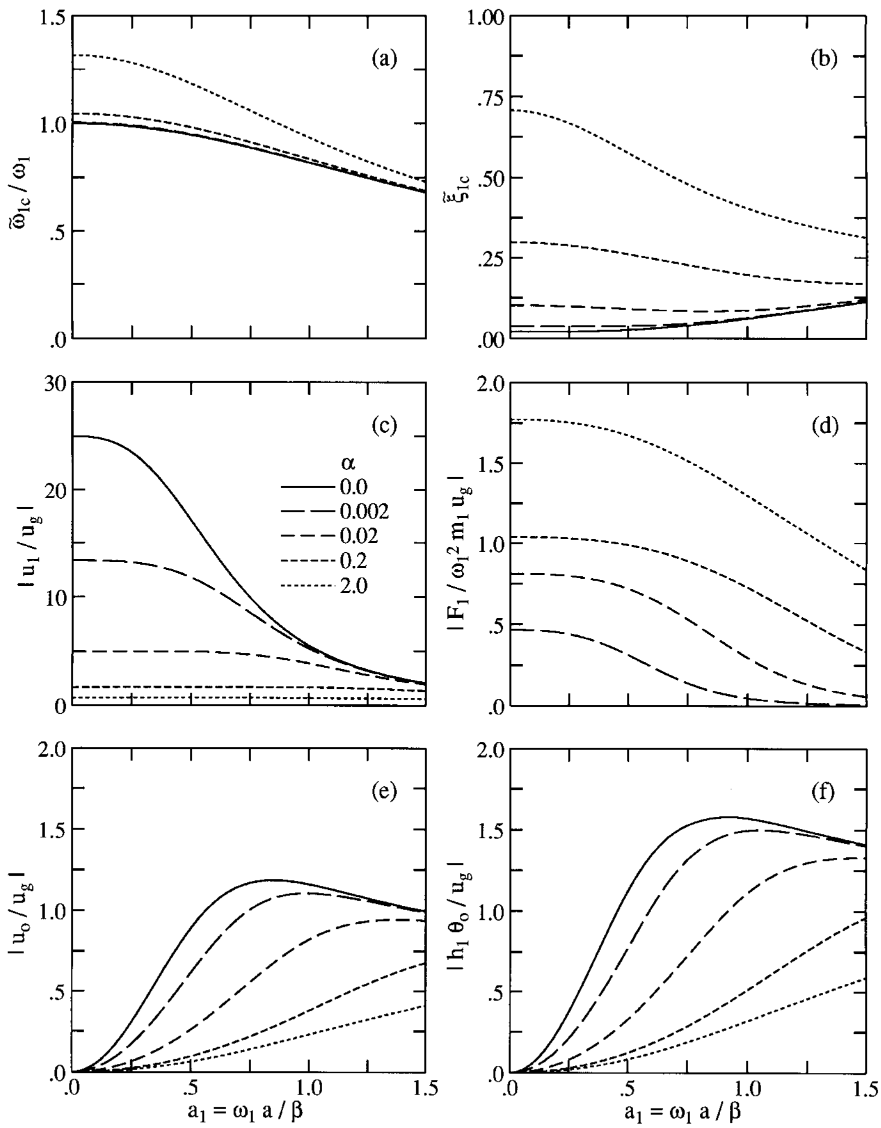

Figure 7. Effects of SSI on the response of an actively controlled one-storey structure for the case of gains determined with SSI. Results include: (a) $\tilde{\omega}_{1c}/\omega_1$, (b) $\tilde{\xi}_{1c}$ and (c) $|u_1/u_g|$, (d) $|F_1/\omega_1^2 m_1 u_g|$, (e) $|u_0/u_g|$ and (f) $|h_1\theta_0/u_g|$ at $\omega = \tilde{\omega}_{1c}$. Slenderness ratio $h_1/a = 1{\cdot}0$

The amplitude of the internal control force F_1 at $\omega = \tilde{\omega}_{1c}$ corresponds to

$$\left| \frac{F_1}{\omega_1^2 m_1 u_g} \right| = \frac{1}{2\tilde{\xi}_{1c}} \sqrt{\bar{g}^2 + \left(\frac{\tilde{\omega}_{1c}}{\tilde{\omega}_1} \right)^2 \bar{h}^2} \tag{61}$$

where $\bar{g}$ and $\bar{h}$ are given by equations (52) and (53), respectively.

A second and preferable approximate approach to obtain the response of the controlled structure including soil–structure interaction effects is to substitute the internal control force $F_1 = (\omega_1/\tilde{\omega}_1)^4 \tilde{F}_1$ in which $\tilde{F}_1$ is given by equation (51) into equation (3), the equation of motion for the top mass. The resulting equation of motion

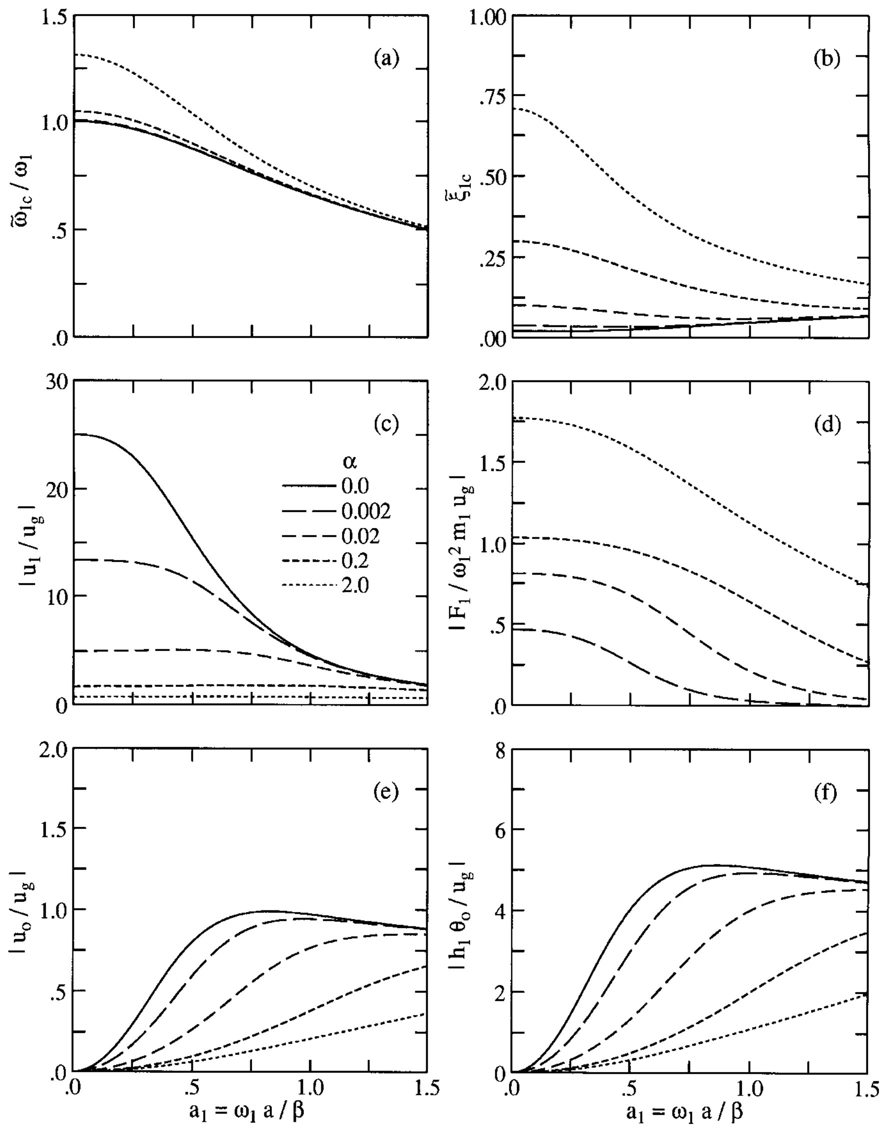

Figure 8. Effects of SSI on the response of an actively controlled one-storey structure for the case of gains determined with SSI. Results include: (a) $\tilde{\omega}_{1c}/\omega_1$, (b) $\tilde{\xi}_{1c}$ and (c) $|u_1/u_g|$, (d) $|F_1/\omega_1^2 m_1 u_g|$, (e) $|u_0/u_g|$ and (f) $|h_1\theta_0/u_g|$ at $\omega = \tilde{\omega}_{1c}$. Slenderness ratio $h_1/a = 2{\cdot}0$

is given again by

$$m_1(\ddot{u}_0 + h_1\ddot{\theta}_0 + \ddot{u}_1) + 2m_1\omega_{1c}\xi_{1c}\dot{u}_1 + m_1\omega_{1c}^2 u_1 = -m_1\ddot{u}_g \tag{62}$$

in which now

$$\omega_{1c} = \omega_1\sqrt{1+g} \tag{63}$$

and

$$\xi_{1c} = \left(\frac{\omega_1}{\omega_{1c}}\right)\left[\xi_1 + \frac{1}{2}h\right] \tag{64}$$

Thus, the superstructure subjected to an internal control force F_1 can be represented by an equivalent 1-DOF structure with frequency ω_{1c} and damping ratio ξ_{1c}. The response of this equivalent structure to the seismic excitation in the presence of soil–structure interation effects can then be obtained by using equation (41) in which $\tilde{\omega}_{1c}$ and $\tilde{\xi}_{1c}$ are calculated from equations (42) and (43) with ω_{1c} and ξ_{1c} given by equations (63) and (64). The values of the transfer functions $|u_0/u_g|$, $|h_1\theta_0/u_g|$ and $|u_1/u_g|$ at the controlled system frequency $\omega=\tilde{\omega}_{1c}$ are again given by equations (44a)–(44c). The amplitude of the control force at $\tilde{\omega}_{1c}$ is also given by equation (45) where g, h, ω_{1c}, ξ_{1c}, $\tilde{\omega}_{1c}$, and $\tilde{\xi}_{1c}$ are now calculated by using equations (55), (56), (63), (64), (42) and (43), respectively. This second approach is used in the calculations that follow.

The effects of SSI on the seismic response of a one-storey structure subjected to active control by internal forces with gains determined including SSI effects are shown in Figures 7 and 8. Again, the characteristics of the structure, foundation and control system correspond to $m_1/\rho a^3=1{\cdot}0$, $h_1/a=1$ (Figure 7) and $h_1/a=2$ (Figure 8), $\xi_1=0{\cdot}02$, $\xi_s=0{\cdot}02$, $v=1/3$ and $\gamma_1=\gamma_2=1$.

The results in Figures 7 and 8 for the case of gains determined including SSI effects follow the same trends as those shown in Figures 4 and 5 for the case of gains determined without SSI. For $a_1>0.5$, the peak values of the transfer fuctions u_1/u_g, u_0/u_g and $h_1\theta_0/u_g$ are slightly larger and the control force is slightly smaller when the SSI effects are included in the calculation of the gains.

CONCLUSIONS

A simple model for the seismic response of a one-storey structure subjected to active control in the presence of soil–structure interaction effects has been presented. The resulting representation in terms of a modified 1-DOF oscillator which includes the effects of control and soil–structure interaction offers insight into the problem and allows us to evaluate the effects of interaction on the seismic response of actively controlled structures in which the control gains have been determined with and without inclusion of SSI effects.

It has been found that control reduces not only the internal deformation of the structure but also the relative horizontal displacement and the rocking motion of the base. If the control forces are small, the effects of SSI tend to reduce the deformation of the structure and the control forces while increasing the relative displacement and rocking motion of the base. For large control forces, the SSI effects may lead to deformations of the structure slightly larger than those obtained when the interaction effects on the response are ignored. The effects of ignoring the interaction between the structure and the soil in the calculation of the control gains are small and result in an slightly lower response of the structure and the foundation at the expense of a slightly larger control force.

The present results for internal control are slightly less favorable than those found by Wong and Luco[3] for an instance of external control. Similar advantages have been obtained by Wu and Smith[9] in their comparison of externally and internally controlled systems.

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